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A GEOMETRICAL MODEL REGARDING THE ENGENDERING GENERALIZATION OF SOME POLYFORM SURFACES

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ABSTRACT

In this paper, it is presented an engendering model by wrapping of a polyform conic - helicoid surface, using as a generatrix a cylindrical surface of revolution.

The model allows, by details (specific features), the determination of more know – types of polyform surfaces and also creates the pre – requisites for the image of a new shape of such a surface.

1. Introduction

There are known [1] a lot of the engendering possibilities of the complex surface that may be included in the polyform surfaces class. Different modalities of analytic modeling of these surfaces create the pre – requisites for the study of the features of this kind of structures. As it follows, it is proposed a general model from the engendering, by wrapping this surfaces with a ended corps by a peripherical primary cylindrical surface of revolution.

The modeling may be done calling on to the general principles of the surfaces engendering, on the basis of the general method GOHMAN, so that. the engendering of the polyform surfaces by known methods may be regarded as expressing particular cases of this complex way of engendering.

2. The reference systems. The engendering surface

In fig. 1, there are defined the referential systems towards there are related the engendering surface (polyform surface) and also the engendering corps ended by a cylindrical surface of revolution.

There are defined the following reference systems:

- $X_1Y_1Z_1$ - the system that is in solidarity with the peripherical surface of the engendering corps;
- $\xi\eta\zeta$ - the solidar system with the surface that should be engendered (the half finished surface – the polyform shaft);
- $X_0Y_0Z_0$ – the mobile solidar system of reference, in the rotation movement with the system $\xi\eta\zeta$;
- XYZ - the mobile solidar system of reference, in the rotation movement with $X_1Y_1Z_1$;
- xyz and $x_0y_0z_0$ - stabile systems of reference, having the axles z and z_0 united with the rotation axles.

The engendering corps – C – has its axle inclined in comparison with the axle of the shaft (the polyform surface) and it does. towards it, two movements:

- of translation, in a paralel direction with the axle of the surface that should be engendered, of size $p\varphi_1$. where p is the helicoidally parameter;

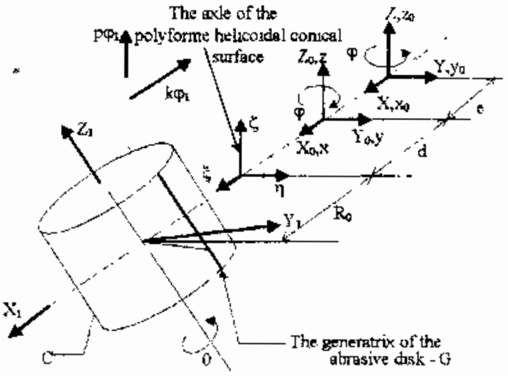

Fig. 1 Systems of reference

- of translation, in a radial direction towards the worked – material, of size $k\varphi_1$, where k is parameter of an archimedean spiral.

In the first picture there were defined the angular parameters of rotation of the systems $X_0Y_0Z_0$ and XYZ surrounding the axles of rotation z_0 and z, i.e. φ_1 and φ_2.

The generating corps C, imagined as a cylindrical surface of revolution is generated by the rectilinear generatrix G:

$$G\begin{vmatrix} X_1 = 0; \\ Y_1 = R; \\ Z_1 = u, \end{vmatrix} \quad (1)$$

that, in the movement,

$$\begin{Vmatrix} X_1 \\ Y_1 \\ Z_1 \end{Vmatrix} = \begin{Vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 0 \\ R \\ u \end{Vmatrix} \quad (2)$$

determines the parametrical equations of the peripherical surface of the generating corps C.

After the developments, we obtain:

$$C\begin{vmatrix} X_1 = R\cos\theta; \\ Y_1 = R\sin\theta; \\ Z_1 = u, \end{vmatrix} \quad (3)$$

where u and θ they are variable parameters.

Obviously, other from of the generatrix G brings us to a surface of revolution different from (3) but with a higher degree of generalisation.

3. Movements of engendering. The family of surfaces

Here are defined, in correlation with the first picture, the next engendering movements:

- the rotation of the system XYZ, surrounding the axel z_0, of an angular parameter φ_1;

$$x_0 = \omega_3^T(\varphi_1)X; \quad (4)$$

- the movement of rotation of the system $X_0Y_0Z_0$, towards xyz, of the angular parameter φ_2, surrounding the axel z;

$$x = \omega_3^T(-\varphi_2)X_0; \quad (5)$$

- the relative position of the systems XYZ and $X_1Y_1Z_1$ solidary with the movement of rotation,

$$X = \beta[X_1 - k(\varphi_1)], \quad (6)$$

where $k(\varphi_1) = \|(-R_0 + k\varphi_1) \quad 0 \quad p\varphi_1\|^T$ (7)

$$\text{and } \beta = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{Vmatrix}; \quad (8)$$

- the relative position of the unchanged systems of reference,

$$x = x_0 - e \quad (9)$$

with $e = \|e \quad 0 \quad 0\|^T$; (10)

- the relative position of the systems ξηζ and $X_0Y_0Z_0$

$$\xi = X_0 - d, \quad (11)$$

with $d = \|d \quad 0 \quad 0\|^T$. (12)

The size R_0, d, e, p, k are constructive size.

By replacing the matrix X_0 form (5) is being determined

$$\xi = \omega_3(-\varphi_2)x - d \quad (13)$$

where, according to (9), results

$$\xi = \omega_3(-\varphi_2)[x_0 + e] - d. \quad (14)$$

Finally, replacing X_0 with the form (4) is being defined the movement

$$\xi = \omega_3(-\varphi_2)\{\omega_3^T(\varphi_1)X + e\} - d. \quad (15)$$

This form, taking into account the relation (6) it becomes

$$\xi = \omega_3(-\varphi_2)\{\omega_3^T(\varphi_1)\beta[X_1 - k(\varphi_1)] + e\} - d \quad (16)$$

representing the movement of a point from the system ξηζ of the generated surface, towards the engendering corps (the polyform helicoidal conical surface).

The "opposite" of the movement (16) may be defined in the form:

$$X_1 = \beta^T\omega_3(\varphi_1)\{\omega_3^T(-\varphi_2)[\xi + d] - e\} - k(\varphi_1) \quad (17)$$

representing the movement of a point from the system ξηζ of the generated surface, towards the generator (corps).

From the relations (16) and (3) can be determined the equation of the family of surfaces C (from the generator) in the reference system of the generated surface.

$$\begin{Vmatrix} \xi \\ \eta \\ \zeta \end{Vmatrix} = \begin{Vmatrix} \cos\varphi_2 & -\sin\varphi_2 & 0 \\ \sin\varphi_2 & \cos\varphi_2 & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot$$

$$\cdot\left\{\begin{Vmatrix} \cos\varphi_1 & -\sin\varphi_1 & 0 \\ \sin\varphi_1 & \cos\varphi_1 & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{Vmatrix} \cdot\right.$$

$$\left.\cdot\left[\begin{Vmatrix} R\cos\theta \\ R\sin\theta \\ u \end{Vmatrix} - \begin{Vmatrix} -R_0 - k\varphi_1 \\ 0 \\ p\varphi_1 \end{Vmatrix}\right] - \begin{Vmatrix} e \\ 0 \\ 0 \end{Vmatrix}\right\} - \begin{Vmatrix} d \\ 0 \\ 0 \end{Vmatrix}. \quad (18)$$

Theoretically, the equations (18) are dependent on the variables φ_1, φ_2, θ, u in form:

$$(C)\varphi_1\begin{vmatrix} \xi = \xi(\varphi_1, \varphi_2, \theta, u); \\ \eta = \eta(\varphi_1, \varphi_2, \theta, u); \\ \zeta = \zeta(\varphi_1, \varphi_2, \theta, u) \end{vmatrix} \quad (19)$$

It may be established a correlation between the angular parameters φ_1 and φ_2 so as the rapport of transmission may be defined,

$$\varphi_2 = i\varphi_1. \tag{20}$$

The evolutes of the family of surfaces (19) represents the generated surface – **the poliforme helicoidal – conical surface.**

The evolute of the family (19) is determined by associating to these equations (19) the condition of wrapping.

4. The condition of wrapping

The condition GOHMAN (see (6)) for the determination of the evolutes of a family of surfaces generated as a result of a kinematics process is:

$$\vec{N}_C \cdot \vec{R}_{\varphi_1} = 0 \tag{21}$$

where: $\vec{N}_C$ is the normal at the surface of the generatrix (see relation (3));

$\vec{R}_{\varphi_1}$ is the speed vector in the relative movement of point belonging to the space $\xi\eta\zeta$ of the surface to be generated, towards $X_1Y_1Z_1$ the space associated to the generating corps.

It is calculated, from the (17),

$$R\varphi_1 = \frac{dX_1}{d\varphi_1} \tag{22}$$

or

$$R_{\varphi_1} = \beta^T \dot{\omega}_3(\varphi_1)\left\{\omega_3^T(-\varphi_2)[\xi + d] + e\right\} - \dot{k}(\varphi_1) + i\beta^T \omega_3(\varphi_1)\dot{\omega}_3^T(-\varphi_2)\frac{d\varphi_2}{d\varphi_1}[\xi + d] \tag{23}$$

Replacing the expression of ξ from the relation (16) in (23) results the form

$$R_{\varphi_1} = \beta^T\dot{\omega}_3(\varphi_1)\left\{\omega_3^T(-\varphi_2)\left[\omega_3(-\varphi_2)\left(\omega_3^T(\varphi_1)\beta(X_1+k) - e\right)\right] + e\right\} - \dot{k}(\varphi_1) + i\beta^T\omega_3(\varphi_1)\cdot\dot{\omega}_3^T(-\varphi_2)\left[\omega_3(-\varphi_2)\left(\omega_3^T(\varphi_1)\beta(X_1 + k(\varphi_1)) - e\right)\right]. \tag{24}$$

that, after the development, become

$$R_{\varphi_1} = \beta^T\left\{\left[\dot{\omega}_3(\varphi_1)\cdot\omega_3^T(-\varphi_2)\cdot\omega_3(-\varphi_2) + i\omega_3(\varphi_1)\cdot\dot{\omega}_3^T(-\varphi_2)\cdot\omega_3(-\varphi_2)\right]\cdot\left[\omega_3^T(\varphi_1)\beta[X_1 + d] - e\right] - \dot{\omega}_3(\varphi_1)e - \beta\dot{k}(\varphi_1)\right\} \tag{25}$$

Finally, we arrive to the expression

$$RM_1 = \begin{Vmatrix} (1-i)Y_1\cos\omega + (1-i)[Z_1 + p\varphi_1]\sin\omega + ie\sin\varphi_1 + k \\ -(1-i)[X_1 - R_0 + a\varphi_1]\cos\omega + ie\cos\varphi_1 + p\sin\omega \\ -(1-i)[X_1 - R_0 + a\varphi_1]\sin\omega - p\cos\omega \end{Vmatrix} \tag{26}$$

of the vector having the speed direction in the relative movement of the mobile systems of reference.

Taking into account the shape of the C (3) surface, is calculated the normal of this,

$$\vec{N}_c = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -R\sin\theta & R\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = R\cos\theta\vec{i} + R\sin\theta\vec{j}. \tag{27}$$

It results, from (3), (21), (26) and (27), the condition of wrapping, in form

$$\left\{(1-i)R\sin\theta\cos\omega + (1-i)[u + p\varphi_1]\sin\omega + ie\sin\varphi_1 + k\right\}\cos\theta + \left\{-(1-i)[R\cos\theta - R_0 + a\varphi_1]\cos\omega + ie\cos\varphi_1 + p\sin\omega\right\}\sin\theta = 0 \tag{28}$$

that, theoretically, should be examined as a dependence

$$\phi(\theta, u, \varphi_1) = 0. \tag{29}$$

The ensemble of equations (19), (29), represents the evolutes of the family of surfaces C, in its relative movement, towards the reference system $\xi\eta\zeta$, i.e. the polyform helicoidal – conical surface.

Note

This generated surface hasn't any signification for the practical utilizations than in particular situations, so as, the analytical model presented here is used for making evident some particular cases, making up a generalisation of the kinematics models of engendering of the polyform surfaces.

5. Particular cases of engendering

a. A polyform surface generated with a circular generatrix

In this case, the kinematics elements are simplifical this way:

- p=0 (p – the helicoidal parameter);
- k=0 (k – the volute parameter);
- ω=0 (the axles of the generatrix surface C and of the generated surface are parallel);
- d=0.

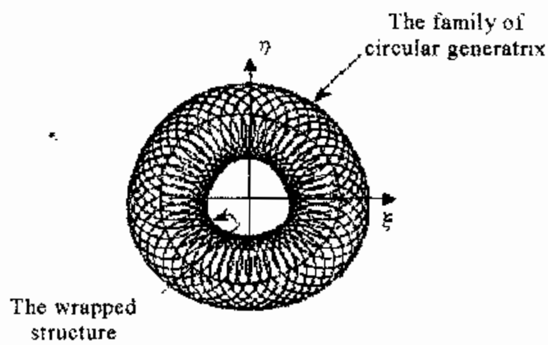

Fig. 2 The polyform structure (3/4)

In this case, the vector $\vec{R}_{\varphi_1}$, becomes:

$$R_{\varphi_1} = \begin{Vmatrix} (1-i)Y_1 + ie\sin\varphi_1 \\ -(1-i)[X_1 - R_0] + ie\cos\varphi_1 \\ 0 \end{Vmatrix} \quad (30)$$

In fig. 2, is theoretically presented the polyform structure resulted.

b. The comprehensive polyform surface

Particular elements of the generalized model:

p=0;
ω=0;
k=0;
d=0;
e=0.

In fig. 3, is presented a polyform and comprehensive structure generated in accordance with the presented scheme

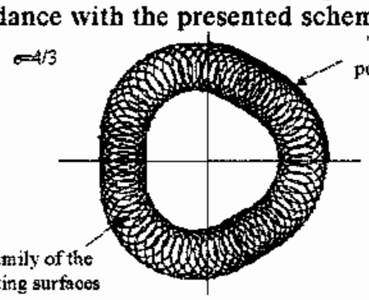

Fig. 3 The generated polyform structure (3/4)

c. The polyform surface generated with a rectilinear generatrix

The case is particularized by:

p=0;
ω=0;
k=0;
d=0;

In fig. 4 is presented a polyform structure generated with a rectilinear generatrix

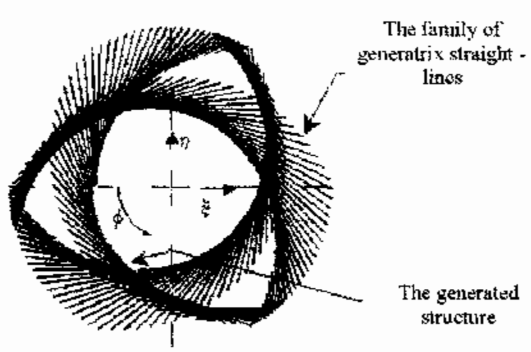

Fig. 4 The form of the generated structure (3/4)

The polyform surface having a point as a generatrix, situation which corresponds to the engendering by lathing this kind of surface, it also may be obtained in the previous paragraph, for the conditions:

R=0;
p=0;
k=0;
ω=0;
d=0;
u=0,

which leads to equations of the form:

$\xi = R_0\cos(\varphi_1+\varphi_2) - e\cos\varphi_2$;
$\eta = R_0\sin(\varphi_1+\varphi_2) - e\sin\varphi_2$;
$\zeta = 0$.

Conclusions

The general analytical model presented here has the quality to permit by detailing the technological constants of the model – the passing to unknown types of polyform structures [2], [3], [4], [5].

Moreover, may create the premises of innovation of this kind of surfaces, by changing the type of generatrix surface but also by facing – following the model –some kinematics schemes of particular engendering.

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MODEL GEOMETRIC PRIVIND GENERALIZAREA GENERĂRII UNOR SUPRAFEȚE POLIFORME

REZUMAT

În lucrare, se prezintă un model al generării prin înfăşurare al unei suprafețe poliformă-elicoidal-conică, utilizând ca generatoare o suprafață cilindrică de revoluție.

Modelul permite, prin particularizări, determinarea mai multor tipuri cunoscute de suprafețe poliforme şi, de asemenea, creează premisele pentru imaginea unei forme noi de astfel de suprafețe.

UN MODÈLE GÉOMÉTRIQUE CONCERNANT LA GÉNÉRALISATION DE CRÉATION D'UN CERTAIN POLYFORM APPRÊTE

ABSTRAIT

En cet article, il est présenté un modèle de création par l'emballage d'un polyform conique. surface helicoid, en utilisant comme génératrice une surface cylindrique de révolution.

Le modèle permet, par des détails (dispositifs spécifiques), la détermination de plus savent. les types de polyform apprête et crée également pré. conditions requises pour l'image d'une nouvelle forme d'une telle surface.

EIN GEOMETRISCHES MODELL BETREFFEND IST DIE ERZEUGENCVERALLGEMEINERUNG IRGENDEINES POLYFORM SURFACES

AUSZUG

In diesem Papier, wird es ein Erzeugenmodell dargestellt. indem man eines konischen polyform aufwickelt. helicoid Oberfläche mit als Erzeugerin einer zylinderförmigen Oberfläche der Umdrehung.

Das Modell erlaubt. durch Details (spezifische Eigenschaften), die Ermittlung von mehr wissen. Arten von polyform surfaces und erstellt auch vor. Erfordernisse für das Bild einer neuen Form solch einer Oberfläche.